Use Newton's method to approximate a solution to $2\cos x = 3x$. (Let $x_0 = \frac{\pi}{6}$ and find $x_2$.)

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$2\cos x = 3x$, $x_0 = \frac{\pi}{6}$

Let $f(x) = 2\cos x - 3x$.

Finding a solution to $f(x) = 0$,

$2\cos x - 3x = 0$

$f'(x) = -2\sin x - 3$

$f\left(\frac{\sqrt{3}}{4} - \frac{\pi}{24}\right)$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= \frac{\pi}{6} - \frac{2\left(\frac{\sqrt{3}}{2}\right) - 3\left(\frac{\pi}{6}\right)}{-2\left(\frac{1}{2}\right) - 3}$$

$$= \frac{\pi}{6} - \frac{\sqrt{3} - \frac{\pi}{2}}{-4}$$

$$= \frac{\pi}{6} + \frac{\sqrt{3}}{4} - \frac{\pi}{8}$$

$$= \frac{4\pi + 6\sqrt{3} - 3\pi}{24}$$

$$= \frac{\pi}{24} + \frac{\sqrt{3}}{6}$$

$$= 0.5639$$

$f\left(\frac{\sqrt{3}}{4} - \frac{\pi}{24}\right)$

$= 2\cos\left(\frac{\sqrt{3}}{4} - \frac{\pi}{24}\right) - 3\left(\frac{\sqrt{3}}{4} - \frac{\pi}{24}\right)$

$= -0.001396$

$f'\left(\frac{\sqrt{3}}{4} - \frac{\pi}{24}\right)$

$= -2\sin\left(\frac{\sqrt{3}}{4} - \frac{\pi}{24}\right) - 3$

$= -4.06899$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= \frac{\sqrt{3}}{4} - \frac{\pi}{24} - (0.000343)$$

$$= 0.5636$$